NASA Technical Memorandum 81694

Lubrication of Rigid Ellipsoidal Solids

(NASA-TM-81694) LUBRICATION OF RIGID ELLIPSIDA SOLIDS (NASA) 55 p HC A04/MF A01 CSCL 13I
N82-25518
Unclas
G3/37 28023

Bernard J. Hamrock
Lewis Research Center
Cleveland, Ohio

and

Duncan Dowson
The University of Leeds
Leeds, England

May 1982

NASA

NASA Technical Memorandum 81694

Lubrication of Rigid Ellipsoidal Solids

Bernard J. Hamrock
Lewis Research Center
Cleveland, Ohio

and

Duncan Dowson
The University of Leeds
Leeds, England

NASA
National Aeronautics and Space Administration
Lewis Research Center

1982

LUBRICATION OF RIGID ELLIPSOIDAL SOLIDS

There have been relatively few theoretical studies of the lubrication of rolling contacts compared with the attention given to plain thrust and journal bearings. Certainly a great deal of development effort has been put into gear, ball, and roller bearing lubrication since all are machine elements of great technological and commercial importance. But understanding of their lubricating mechanism is still far from complete. The huge pressures involved in the elliptical contact generated between a rolling element and a track influence in a rather complicated way the properties of the lubricant and the shapes of the contacting surfaces.

The first requirement therefore is to develop a basic solution to the problem of the lubrication of rigid ellipsoidal solids with an isoviscous, incompressible fluid. A solution to this problem is presented in this chapter, and the results provide a foundation for the full appreciation of the elastohydrodynamic theory of elliptical contacts to be presented later. The influence of the variation of viscosity with pressure and

*Published as Chapter 6 in Ball Bearing Lubrication by Bernard J. Hamrock and Duncan Dowson, John Wiley & Sons, Inc., Sept. 1981.

the effect of the deformation of surfaces under pressure are considered in Chapter 7.

Most studies of the hydrodynamic lubrication of nominal point and line contacts have concentrated on minimum-film-thickness predictions for either a ball near a plane or a cylinder near a plane in which side leakage is neglected. But the full range of geometries between the two extremes has only recently been studied by Brewe, et al. (1979). Kapitza (1955) presented an early and elegant analysis of this problem in which he generated a minimum-film-thickness formula for a nonrotating sphere floating in a sea of lubricant above a plane surface sliding at a given velocity. This analytical solution represents a rare and outstanding example of a successful mathematical approach to the solution of the second-order differential equation presented by Osborne Reynolds. Solutions taking account of side leakage were developed analytically for the special case of a ball on a plane by using a clever substitution. One disadvantage of the Kapitza approach is that it adopted the half-Sommerfeld boundary condition, which violates the requirement of flow continuity at the cavitation boundary. However, the effect of this assumption on film thickness prediction is not always serious. The Brewe, et al. (1979) work is used extensively in this chapter because it uses the Reynolds cavitation boundary conditions and is applicable to the complete range of geometries of the contacting solids.

The material presented in this chapter not only provides an introduction to classical hydrodynamic lubrication as it relates to nonconformal contacts, but it also introduces a minimum-film-thickness formula that has a number of direct applications: circular-arc thrust bearing pads; industrial coating processes in which paint, emulsion, or protective coatings are applied to sheet or film materials passing betweens rollers; and very lightly loaded cylindrical roller bearings.

6.1 Equation for Pressure Distribution

From the previous chapter the appropriate form of the Reynolds equation when the viscosity is assumed to be constant is given as

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 12u\eta_0 \frac{\partial h}{\partial x} \qquad (5.45)$$

where

$$u = \frac{u_a + u_b}{2}$$

It is convenient to nondimensionalize with respect to the effective radius R_x; that is,

$$X = \frac{x}{R_x}, \qquad Y = \frac{y}{R_x}, \qquad H = \frac{h}{R_x}, \qquad \text{and} \quad P = \frac{pR_x}{\eta_0 u} \qquad (6.1)$$

In terms of these dimensionless variables equation (5.45) becomes

$$\frac{\partial}{\partial X}\left(H^3 \frac{\partial P}{\partial X}\right) + \frac{\partial}{\partial Y}\left(H^3 \frac{\partial P}{\partial Y}\right) = 12 \frac{\partial H}{\partial X} \qquad (6.2)$$

The film thickness between two rigid ellipsoidal solids as shown in Figure 2.18 can be written as

$$h = h_0 + S \qquad (6.3)$$

where

S = geometrical separation of the solids

h_0 = central (minimum) film thickness

Using equation (2.35) and rewriting equation (6.3) in dimensionless form give

$$H = H_0 + \frac{X^2}{2} + \frac{Y^2}{2\alpha_a} \qquad (6.4)$$

where

$$\alpha_a = \frac{R_y}{R_x} \qquad (6.5)$$

$$H_0 = \frac{h_0}{R_x} \qquad (6.6)$$

In equation (6.4) the parabolic approximation is used in defining the geometrical separation of the undeformed solids.

The solution of the Reynolds equation (6.2) consists of a homogeneous and a particular solution; that is,

$$P = P_h + P_p \qquad (6.7)$$

for which P_h is a solution to the homogeneous equation and the condition that $P_h = -P_p$ at the boundaries leads to

$$\frac{\partial}{\partial X}\left(H^3 \frac{\partial P_h}{\partial X}\right) + \frac{\partial}{\partial Y}\left(H^3 \frac{\partial P_h}{\partial Y}\right) = 0 \qquad (6.8)$$

Kapitza (1955) was the first to recognize that the particular solution for the pressure is simply proportional to X/H^2, or

$$P_p = -\frac{4\lambda_b X}{H^2} \qquad (6.9)$$

where

$$\lambda_b = \frac{1}{1 + 2/3\alpha_a} \qquad (6.10)$$

In the preceding equation λ_b is the side-leakage factor established by Archard and Cowking (1965-66) and can be verified by inserting P_p back into equation (6.2). If we define $P_h(X,Y) = 4\lambda_b Q(X,Y)$ by using equation (6.7), we can express the full solution as

$$P = 4\lambda_b \left(-\frac{X}{H^2} + Q\right) \qquad (6.11)$$

In general the homogeneous solution P_h is an unknown function of X and Y. Consequently, the pressure distribution must be determined numerically for a given speed, viscosity, geometry, and film thickness. The numerical solution is

normally achieved by the Gauss-Seidel iterative method with overrelaxation.

A variable-mesh nodal structure like the one shown in Figure 6.1 is used to provide close spacing in and around the pressure peak. In this figure the inlet is to the left and the outlet is to the right. The variable mesh helps to minimize the errors that can occur because of large gradients in the high-pressure region. The grid spacing of the coordinates X and Y varies depending on the anticipated pressure distribution. That is, for a very highly peaked and localized pressure distribution, the dimensionless fine mesh spacing is normally about 0.002, and the coarse mesh about 0.1. For a relatively flat pressure distribution, the fine mesh would be about 0.005, and the coarse mesh about 0.13.

6.2 Boundary Conditions

At the inlet the pressure is taken as zero at a distance sufficiently far from the center of the contact to give fully flooded conditions. Similarly, at a sufficient distance from the center of the contact to the sides of the nodal structure, the pressure is also taken to be zero. The boundary condition in the outlet is not as straightforward since it is necessary to take account of cavitation.

In the outlet region there is a tendency to form subambient pressures, which lead to disruption of the lubricating film by cavitation. The usual form of cavitation in lubricating films is the liberation of dissolved gases. Mineral oils contain between 8 and 10 percent of dissolved air. When the pressure in the oil film falls below ambient, some of the air is liberated in the form of bubbles. This tends to maintain the oil film pressure near the level of the saturation pressure. For most lubrication conditions the saturation and ambient pressures will be almost equal. These observations suggest that the pressure in the cavitated region of lubricating films will be approximately constant and near to the atmospheric or ambient pressure.

The approach adopted by Kapitza (1955) in defining the outlet boundary condition was to ignore the negative pressures, that is, to employ the half-Sommerfeld boundary condition. Kapitza's solution has the appeal of simplicity, but it does not satisfy the flow continuity conditions at the cavitation boundary, namely, that the pressure gradient normal to the cavitation boundary must be zero. To insist that $P = \partial P/\partial N = 0$ at $X = 0$ would be overspecifying the problem mathematically. It is possible, however, to insist that $P = \partial P/\partial N = 0$ at the cavitation boundary and to locate the interface in such a position that this well-known Reynolds boundary condition is satisfied.

6.3 Load Capacity

Once the pressure distribution for the appropriate cavitation boundary conditions has been determined numerically, we can express the load capacity as

$$F = \iint p \, dx \, dy \tag{6.12}$$

By making use of equation (6.11) we can write this equation in dimensionless form as

$$F = 4\eta_0 u R_x \lambda_b \iint \left(- \frac{X}{H^2} + Q \right) dX \, dY \tag{6.13}$$

The central film thickness can be isolated from the integrand by introducing the following transformations:

$$\left. \begin{aligned} X &= X_t (2H_0)^{1/2} \\ Y &= Y_t (2\alpha_a H_0)^{1/2} \end{aligned} \right\} \tag{6.14}$$

If we assume the homogeneous solution to transform in the same manner as the particular solution, we obtain

$$F = 8\lambda_b \eta_0 u R_x \left(\frac{2\alpha_a}{H_0} \right)^{1/2} \iint \left[\frac{-X_t}{\left(1 + X_t^2 + Y_t^2\right)^2} + Q(X_t, Y_t) \right] dX_t \, dY_t$$

(6.15)

Kapitza (1955) refers to this integral as the reduced hydrodynamic lift L_t. Thus

$$L_t = \iint \left[\frac{-X_t}{\left(1 + X_t^2 + Y_t^2\right)^2} + Q(X_t, Y_t) \right] dX_t \, dY_t \qquad (6.16)$$

6.4 Film Thickness Formula

The reduced hydrodynamic lift was found by Kapitza (1955) to equal $\pi/2$ by assuming $Q = 0$ and integrating over the half-space of positive pressures. For the Reynolds boundary conditions the limits depend on the shape of the cavitated region and hence on the geometry. Consequently we seek an additional geometrical effect to modify and generalize Kapitza's solution. The central (minimum) film thickness can be expressed as a function of the load, speed, geometry, and fluid viscosity by rearranging equation (6.15) and writing

$$H_{min} = H_0 = 128 \, \alpha_a \left(\frac{\lambda_b \eta_0 u R_x}{F} L_t \right)^2 \qquad (6.17)$$

The ratio of dimensionless speed to dimensionless load can be defined as

$$\frac{U}{W} = \frac{\eta_0 u R_x}{F} \qquad (6.18)$$

and equation (6.17) becomes

$$H_{min} = H_0 = 128\ \alpha_a \left(\lambda_b L_t \frac{U}{W}\right)^2 \qquad (6.19)$$

The integrand of equation (6.16) thus becomes a function of the geometry represented by α_a and the central film thickness H_0. Consequently $L_t = L_t(\alpha_a, H_0)$, and this results in a transcendental equation for H_0.

It is necessary to determine L_t as a function of the geometry alone, or

$$L_t = L_t(\alpha_a) \qquad \text{if} \quad X_t^2 << \frac{1}{2H_0};\ Y_t^2 << \frac{1}{2H_0} \qquad (6.20)$$

Once the hydrodynamic load-carrying capacity F has been obtained from the numerically determined pressure distribution, L_t can be evaluated for various geometries. A curve fit can then be used to show the effect of geometry on L_t. The following relationship provides a good representation of the numerical results:

$$L_t = 0.131 \tan^{-1}\left(\frac{\alpha_a}{2}\right) + 1.683 \qquad (6.21)$$

Inserting equation (6.21) into equation (6.19) gives the following general expression for minimum film thickness between any rigid ellipsoidal solids, ranging from two spheres in nominal

point contact to infinitely long cylinders in nominal line contact:

$$H_{min} = H_0 = 128\ \alpha_a \left\{ \frac{\lambda_b U}{W} \left[0.131 \tan^{-1} \left(\frac{\alpha_a}{2} \right) + 1.683 \right] \right\}^2 \tag{6.22}$$

6.5 Comparison Between Different Theories for the Lubrication of Rigid Ellipsoidal Solids

The minimum-film-thickness equation derived by Kapitza (1955) using the half-Sommerfeld cavitation boundary condition is

$$(H_{min})_K = 128\ \alpha_a \left(\frac{\pi}{2} \frac{\lambda_b U}{W} \right)^2 \tag{6.23}$$

From equations (6.19) and (6.23) we can write

$$\frac{(H_{min})_K}{H_{min}} = \left(\frac{\pi}{2L_t} \right)^2 \tag{6.24}$$

By using equations (6.21) and (6.24) we find that the film thickness as obtained from equation (6.22) is 11 to 21 percent greater than that obtained from Kapitza's (1955) solution (equation (6.23)), with the smallest difference occurring for a ball-on-plane contact. The alteration of the pressure distribution due to the Reynolds cavitation boundary conditions is responsible for this influence of contact gometry on minimum film

thickness. These differences between the results of Kapitza (1955) and Brewe, et al. (1979) are illustrated graphically in Figure 6.2. For the entire range of the radius ratio R_x/R_y the dimensionless film thickness is higher for the Brewe, et al. (1979) results than for the Kapitza (1955) results.

Both the Kapitza (1955) analysis and the numerical solutions of Brewe, et al. (1979) resulted in an exponent of 2 for U/W in the dimensionless film thickness equation. Dalmaz and Godet (1973) also used the Reynolds boundary conditins for a ball-on-plane configuration, and they reported a comparable exponent of 1.77. This lower exponent has been discussed by Brewe, et al. (1979), and it appears to be due to starvation effects resulting from the inlet boundary condition for both the analytical and experimental (Dalmaz and Godet, 1973) situations. The sophisticated apparatus used by Dalmaz and Godet simultaneously measured the film thickness, traction, and speed between a steel ball and a glass plate in pure sliding. A 30-mm-diameter ball, which turned around one of its axes, was immersed in an oil bath and thus carried the lubricant into the contact area through viscous lifting. An optical interferometric technique was used to measure the oil film thickness.

Experimental results obtained by Dalmaz and Godet (1973) under lightly loaded, isoviscous conditions for pure sliding of a ball on a plane are compared with the theoretical results obtained from equation (6.22) in Figure 6.3. The grouping of

dimensionless parameters in this figure is the same as that used by Dalmaz and Godet (1973) and was first introduced by Thorp and Gohar (1972). The theoretical results obtained from equation (6.22) are in excellent agreement with the experimental data for the lower range of H_0/WG, but at higher speeds the measured film thicknesses fall below the theoretical predictions, probably as a result of lubricant starvation in the inlet region. For comparison, predictions based on the theory of Kapitza (1955) and the calculations of Dalmaz and Godet (1973) have also been included in Figure 6.3.

6.6 Pressure Distribution Between Ellipsoidal Solids

Three-dimensional contour plots of the pressure distribution for values of α_a of 1.00 and 36.54 are presented in Figure 6.4. The shape of the cavitation boundary is clearly evident in this figure. As α_a becomes large, the cavitation boundary tends to straighten out and is accompanied by decreasing changes in L_t. The scale along the Y axis in Figure 6.4 has been magnified about three times to improve the resolution. Consequently the differences in the shapes of the cavitation boundary are actually subdued in Figure 6.4.

Isobar plots for three radius ratios α_a of 25.29, 8.30, and 1.00 are shown in Figure 6.5. The center of contact is represented by an asterisk. The pressure peak builds up in the

entrance region, which is located to the left of the center of contact and is indicated by a cross. Since the isobars in each case are evenly spaced, the pressure gradients can be easily envisaged. Note that, as the radius ratio α_a increases, the steeper pressure gradients are found predominantly in the rolling direction. This implies that the amount of side leakage decreases as α_a increases. A decrease in side leakage is reflected in an increase in the value of λ_b. For nominal line contact $\lambda_b = 1$, and for the largest value of α_a (25.29) recorded in Figure 6.5 the corresponding value of λ_b is 0.974.

6.7 Closure

The influence of geometry on the hydrodynamic lubrication of rigid, ellipsoidal soli. has been investigated in this chapter. The study has been restricted to conjunctions fully immersed in lubricant (i.e., fully flooded). The effect of geometry on film thickness was determined numerically by varying the radius ratio α_a from 1 (a ball-on-plane or ball-on-ball configuration) to 36 (a ball in a conforming groove). Pressure-viscosity effects were not considered. It was found that the minimum film thickness had the same speed, viscosity, and load dependence as found by Kapitza in his classical analytical solution to the problem. When the Reynolds cavitation boundary con-

dition was incorporated in the analysis, an additional geometrical effect was introduced into the film thickness equation. The derived film thickness equations can be compared as follows:

Reynolds boundary condition:

$$H_0 = 128\ \alpha_a \left\{ \frac{\lambda_b U}{W} \left[0.131 \tan^{-1}\left(\frac{\alpha_a}{2}\right) + 1.683 \right] \right\}^2$$

Half-Sommerfeld boundary condition:

$$H_0 = 128\ \alpha_a \left(\frac{\lambda_b U}{W} \frac{\pi}{2} \right)^2$$

With the Reynolds boundary condition the minimum film thickness has been found to be 11 to 21 percent greater than that obtained when using the half-Sommerfeld boundary condition adopted by Kapitza.

SYMBOLS

A	constant used in equation (3.113)
A^*, B^*, C^*, D^*, L^*, M^*	relaxation coefficients
A_v	drag area of ball, m^2
a	semimajor axis of contact ellipse, m
$\bar{a}$	$a/2\bar{m}$
B	total conformity of bearing
b	semiminor axis of contact ellipse, m
$\bar{b}$	$b/2\bar{m}$
C	dynamic load capacity, N
C_v	drag coefficient
$C_1,\ldots,C_8$	constants
c	19,609 N/cm^2 (28,440 lbf/in^2)
$\bar{c}$	number of equal divisions of semimajor axis
D	distance between race curvature centers, m
$\tilde{D}$	material factor
$\bar{D}$	defined by equation (5.63)
De	Deborah number
d	ball diameter, m
$\bar{d}$	number of divisions in semiminor axis
d_a	overall diameter of bearing (Figure 2.13), m
d_b	bore diameter, m
d_e	pitch diameter, m
d_e'	pitch diameter after dynamic effects have acted on ball, m
d_i	inner-race diameter, m
d_o	outer-race diameter, m

E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2\Big/\left(\dfrac{1-\nu_a^2}{E_a}+\dfrac{1-\nu_b^2}{E_b}\right)$, N/m^2
E_a	internal energy, m^2/s^2
$\tilde{E}$	processing factor
E_1	$[(\tilde{H}_{min} - H_{min})/H_{min}] \times 100$
$\mathcal{E}$	elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{E}}$	approximate elliptic integral of second kind
e	dispersion exponent
F	normal applied load, N
F^*	normal applied load per unit length, N/m
$\tilde{F}$	lubrication factor
$\bar{F}$	integrated normal applied load, N
F_c	centrifugal force, N
F_{max}	maximum normal applied load (at $\psi = 0$), N
F_r	applied radial load, N
F_t	applied thrust load, N
F_ψ	normal applied load at angle ψ, N
$\mathcal{F}$	elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{F}}$	approximate elliptic integral of first kind
f	race conformity ratio
f_b	rms surface finish of ball, m
f_r	rms surface finish of race, m
G	dimensionless materials parameter, αE
G^*	fluid shear modulus, N/m^2
$\tilde{G}$	hardness factor
g	gravitational constant, m/s^2

g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
$\hat{H}$	dimensionless film thickness, $H(W/U)^2 = F^2h/u^2\eta_0^2R_x^3$
H_c	dimensionless central film thickness, h_c/R_x
$H_{c,s}$	dimensionless central film thickness for starved lubrication condition
H_f	frictional heat, N m/s
H_{min}	dimensionless minimum film thickness obtained from EHL elliptical-contact theory
$H_{min,r}$	dimensionless minimum film thickness for a rectangular contact
$H_{min,s}$	dimensionless minimum film thickness for starved lubrication condition
$\tilde{H}_c$	dimensionless central film thickness obtained from least-squares fit of data
$\tilde{H}_{min}$	dimensionless minimum film thickness obtained from least-squares fit of data
$\bar{H}_c$	dimensionless central-film-thickness - speed parameter, $H_cU^{-0.5}$
$\bar{H}_{min}$	dimensionless minimum-film-thickness - speed parameter, $H_{min}U^{-0.5}$
$\bar{H}_0$	new estimate of constant in film thickness equation
h	film thickness, m
h_c	central film thickness, m
h_i	inlet film thickness, m

h_m	film thickness at point of maximum pressure, where $dp/dx = 0$, m
h_{min}	minimum film thickness, m
h_0	constant, m
I_d	diametral interference, m
I_p	ball mass moment of inertia, m N s^2
I_r	integral defined by equation (3.76)
I_t	integral defined by equation (3.75)
J	function of k defined by equation (3.8)
J^*	mechanical equivalent of heat
$\bar{J}$	polar moment of inertia, m N s^2
K	load-deflection constant
k	ellipticity parameter, a/b
$\bar{k}$	approximate ellipticity parameter
$\tilde{k}$	thermal conductivity, N/s °C
k_f	lubricant thermal conductivity, N/s °C
L	fatigue life
L_a	adjusted fatigue life
L_t	reduced hydrodynamic lift, from equation (6.21)
$L_1,\ldots,L_4$	lengths defined in Figure 3.11, m
L_{10}	fatigue life where 90 percent of bearing population will endure
L_{50}	fatigue life where 50 percent of bearing population will endure
ℓ	bearing length, m
$\bar{\ell}$	constant used to determine width of side-leakage region
M	moment, Nm

M_g	gyroscopic moment, Nm
M_p	dimensionless load-speed parameter, $WU^{-0.75}$
M_s	torque required to produce spin, N m
m	mass of ball, N s^2/m
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
$\tilde{m}$	dimensionless inlet distance (Figures 7.1 and 9.1)
$\bar{m}$	number of divisions of semimajor or semiminor axis
m_W	dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971)
N	rotational speed, rpm
n	number of balls
n^*	refractive index
$\bar{n}$	constant used to determine length of outlet region
P	dimensionless pressure
P_D	dimensionless pressure difference
P_d	diametral clearance, m
P_e	free endplay, m
P_{Hz}	dimensionless Hertzian pressure, N/m^2
p	pressure, N/m^2
p_{max}	maximum pressure within contact, $3F/2\pi ab$, N/m^2
$p_{iv,as}$	isoviscous asymptotic pressure, N/m^2
Q	solution to homogeneous Reynolds equation
Q_m	thermal loading parameter
$\bar{Q}$	dimensionless mass flow rate per unit width, $q\eta_0/\rho_0 E'R^2$
q_f	reduced pressure parameter
q_x	volume flow rate per unit width in x direction, m^2/s

q_y	volume flow rate per unit width in y direction, m^2/s
R	curvature sum, m
R_a	arithmetical mean deviation defined in equation (4.1), m
R_c	operational hardness of bearing material
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
r	race curvature radius, m
$r_{ax}, r_{bx}, r_{ay}, r_{by}$	radii of curvature, m
r_c, ϕ_c, z	cylindrical polar coordinates
r_s, θ_s, ϕ_s	spherical polar coordinates
$\overline{r}$	defined in Figure 5.4
S	geometric separation, m
S^*	geometric separation for line contact, m
S_0	empirical constant
s	shoulder height, m
T	τ_0/p_{max}
$\widetilde{T}$	tangential (traction) force, N
T_m	temperature, °C
T_b^*	ball surface temperature, °C
T_f^*	average lubricant temperature, °C
ΔT^*	ball surface temperature rise, °C
T_1	$(\tau_0/p_{max})_{k=1}$
T_v	viscous drag force, N
t	time, s
t_a	auxiliary parameter
u_B	velocity of ball-race contact, m/s

u_c	velocity of ball center, m/s
U	dimensionless speed parameter, $\eta_0 u/E'R_x$
u	surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
$\bar{u}$	number of stress cycles per revolution
Δu	sliding velocity, $u_a - u_b$, m/s
v	surface velocity in transverse direction, m/s
W	dimensionless load parameter, $F/E'R^2$
w	surface velocity in direction of film, m/s
X	dimensionless coordinate, x/R_x
Y	dimensionless coordinate, y/R_x
X_t, Y_t	dimensionless grouping from equation (6.14)
X_a, Y_a, Z_a	external forces, N
Z	constant defined by equation (3.48)
Z_1	viscosity pressure index, a dimensionless constant
$x, \tilde{x}, \bar{x}, \bar{x}_1$ $y, \tilde{y}, \bar{y}, \bar{y}_1$ $z, \tilde{z}, \bar{z}, \bar{z}_1$	coordinate system
α	pressure-viscosity coefficient of lubrication, m^2/N
α_a	radius ratio, R_y/R_x
β	contact angle, rad
β_f	free or initial contact angle, rad
β'	iterated value of contact angle, rad
Γ	curvature difference
γ	viscous dissipation, N/m^2 s
$\dot{\gamma}$	total strain rate, s^{-1}
$\dot{\gamma}_e$	elastic strain rate, s^{-1}
$\dot{\gamma}_v$	viscous strain rate, s^{-1}

γ_a	flow angle, deg
δ	total elastic deformation, m
δ^*	lubricant viscosity temperature coefficient, $^{o}C^{-1}$
δ_D	elastic deformation due to pressure difference, m
δ_r	radial displacement, m
δ_t	axial displacement, m
δ_x	displacement at some location x, m
$\bar{\delta}$	approximate elastic deformation, m
$\tilde{\delta}$	elastic deformation of rectangular area, m
ε	coefficient of determination
ε_1	strain in axial direction
ε_2	strain in transverse direction
ζ	angle between ball rotational axis and bearing centerline (Figure 3.10)
ζ_a	probability of survival
η	absolute viscosity at gauge pressure, N s/m^2
$\bar{\eta}$	dimensionless viscosity, η/η_0
η_0	viscosity at atmospheric pressure, N s/m^2
η_∞	6.31×10^{-5} N s/m^2(0.0631 cP)
Θ	angle used to define shoulder height
Λ	film parameter (ratio of film thickness to composite surface roughness)
λ	equals 1 for outer-race control and 0 for inner-race control
λ_a	second coefficient of viscosity
λ_b	Archard-Cowking side-leakage factor, $(1 + 2/3\ \alpha_a)^{-1}$
λ_c	relaxation factor

μ	coefficient of sliding friction
μ^*	$\overline{\rho}/\overline{\eta}$
ν	Poisson's ratio
ξ	divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, s^{-1}
ρ	lubricant density, N s^2/m^4
$\overline{\rho}$	dimensionless density, ρ/ρ_0
ρ_0	density at atmospheric pressure, N s^2/m^4
σ	normal stress, N/m^2
σ_1	stress in axial direction, N/m^2
τ	shear stress, N/m^2
τ_0	maximum subsurface shear stress, N/m^2
$\tilde{\tau}$	shear stress, N/m^2
$\tilde{\tau}_e$	equivalent stress, N/m^2
$\tilde{\tau}_L$	limiting shear stress, N/m^2
Φ	ratio of depth of maximum shear stress to semiminor axis of contact ellipse
Φ^*	$PH^{3/2}$
Φ_1	$(\Phi)_{k=1}$
ϕ	auxiliary angle
ϕ_T	thermal reduction factor
ψ	angular location
ψ_ℓ	limiting value of ψ
Ω_i	absolute angular velocity of inner race, rad/s
Ω_o	absolute angular velocity of outer race, rad/s
ω	angular velocity, rad/s
ω_B	angular velocity of ball-race contact, rad/s
ω_b	angular velocity of ball about its own center, rad/s

ω_c	angular velocity of ball around shaft center, rad/s
ω_s	ball spin rotational velocity, rad/s

Subscripts:

a	solid a
b	solid b
c	central
bc	ball center
IE	isoviscous-elastic regime
IR	isoviscous-rigid regime
i	inner race
K	Kapitza
min	minimum
n	iteration
o	outer race
PVE	piezoviscous-elastic regime
PVR	piezoviscous-rigid regime
r	for rectangular area
s	for starved conditions
x,y,z	coordinate system

Superscript:

$(\bar{\ })$	approximate

ORIGINAL PAGE IS
OF POOR QUALITY

REFERENCES

Abbott, E. J. and Firestone, F. A. (1933) Specifying Surface Quality, Mech. Eng., 55, 569-572.

Agricola, G. (1556) De Re Metallica, Basel.

Aihara, S. and Dowson, D. (1979) "A Study of Film Thickness in Grease Lubricated Elastohydrodynamic Contacts," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics', D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 104-115.

Allan, R. K. (1945) Rolling Bearings, Sir Isaac Pitman & Sons, London.

Alsaad, M., Bair, S., Sanborn, D. M., and Winer, W. O. (1978) "Glass Transitions in Lubricants: Its Relation to Elastohydrodynamic Lubrication (EHD)," J. Lubr. Technol., 100(3), 404-417.

Amontons, G. (1699) "De la resistance caus'ee dans les machines," Memoires de l'Academie Royal, A, Chez Gerard Kuyper, Amsterdam, 1706, 257-282.

Anderson, W. J. (1978) "The Practical Impact of Elastohydrodynamic Lubrication," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 217-226.

Anderson, W. J. and Zaretsky, E. V. (1968) "Rolling-Element Bearings." Mach. Des. (Bearing Reference Issue), 40(14), 22-39.

Anderson, W. J. and Zaretsky, E. V. (1973) "Rolling-Element Bearings - A Review of the State of the Art," Tribology Workshop sponsored by National Science Foundation, Atlanta, Ga., Oct. 19-20, 1972.

ORIGINAL PAGE IS
OF POOR QUALITY

Archard, J. F. (1968) "Non-Dimensional Parameters in Isothermal Theories of Elastohydrodynamic Lubrication." J. Mech. Eng. Sci., 10(2), 165-167.

Archard, J. F. and Cowking, E. W. (1965-66) "Elastohydrodynamic Lubrication at Point Contacts," Proc. Inst. Mech. Eng., London, 180(3B), 47-56.

Archard, J. F. and Kirk, M. T. (1961) "Lubrication at Point Contacts" Proc. R. Soc. London, Ser. A, 261, 532-550.

Archard, J. F. and Kirk, M. T. (1964) "Film Thickness for a Range of Lubricants Under Severe Stress," J. Mech. Eng. Sci., 6, 101-102.

Ausherman, V. K., Nagaraj, H. S., Sanborn, D. M., and Winer, W. O. (1976) "Infrared Temperature Mapping in Elastohydrodynamic Lubrication," J. Lubr. Technol., 98(2), 236-243.

Baglin, K. P. and Archard, J. F. (1972) "An Analytic Solution of the Elastohydrodynamic Lubrication of Materials of Low Elastic Modulus," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 13.

Bair, S. and Winer, W. (1979) "Shear Strength Measurements of Lubricants at High Pressures," J. Lubr. Technol. 101(3), 251-257.

Bamberger, E. N. (1967) "The Effect of Ausforming on the Rolling Contact Fatigue Life of a Typical Bearing Steel," J. Lubr. Technol., 89(1), 63-75.

Bamberger, E. N. (1972) "The Thermomechanical Working of Electro-Slag Melted M-50 Bearing Steel," R72AEG290, General Electric Co., Cincinnati, Ohio.

Bamberger, E. N., Harris, T. A., Kacmarsky, W. M., Moyer, C. A., Parker, R. J., Sherlock, J. J., and Zaretsky, E. V. (1971) Life Adjustment Factors for Ball and Roller Bearings. American Society of Mechanical Engineers, New York.

Bamberger, E. N., Zaretsky, E. V., and Singer, H. (1976) "Endurance and Failure Characteristics of Main-Shaft Jet Engine Bearing at $3x10^6$DN," J. Lubr. Technol., 98(4), 580-585.

Barus, C. (1893) "Isotherms, Isopiestics, and Isometrics Relative to Viscosity," Am. J. Sci., 45, 87-96.

Barwell, F. T. (1974) "The Tribology of Wheel on Rail," Tribol. Int., 7, (4), 146-150.

Barwell, F. T. (1979) "Bearing Systems - Principles and Practice," Oxford University Press, Oxford.

Bell, J. C. and Kannel, J. W. (1970) "Simulation of Ball-Bearing Lubrication with a Rolling-Disk Apparatus," J. Lubr. Technol., 92, 1-15.

Bell, J. C., Kannel, J. W., and Allen, C. M. (1964) "The Rheological Behaviour of the Lubricant in the Contact Zone of a Rolling Contact System," J. Basic Eng., 86(3), 423-432.

Bisson, E. E. and Anderson, W. J. (1964) "Advanced Bearing Technology," NASA SP-38.

Biswas, S. and Snidle, R. W. (1976) "Elastohydrodynamic Lubrication of Spherical Surfaces of Low Elastic Modulus," J. Lubr. Technol., 98(4), 524-529.

Blok, H. (1952) Discussion of paper by E. McEwen. Gear Lubrication Symposium. Part I. The Lubrication of Gears, J. Inst. Petrol., 38, 673.

Blok, H. (1965) "Inverse Problems in Hydrodynamic Lubrication and Design Directives for Lubricated Flexible Surfaces," Proceedings of International Symposium on Lubrication and Wear, D. Muster and B. Sternlicht, eds., McCutchan, Berkeley, 1-151.

ORIGINAL PAGE IS
OF POOR QUALITY

Brewe, D. E., Coe, H. H., and Scibbe, H. W. (1969) "Cooling Studies with High-Speed Ball Bearings Operating in Cool Hydrogen Gas," Trans. ASLE, vol. 12, 66-76.

Brewe, D. E. and Hamrock, B. J. (1977) "Simplified Solution for Elliptical-Contact Deformation Between Two Elastic Solids," J. Lubr. Technol. 99(4), 485-487.

Brewe, D. E., Hamrock, B. J., and Taylor, C. M. (1979) "Effect of Geometry on Hydrodynamic Film Thickness," J. Lubr. Technol., 101(2), 231-239.

Brown, P. F. and Potts, J. R. (1977) "Evaluation of Powder Processed Turbine Engine Ball Bearings," PWA-FR-8481, Pratt & Whitney Aircraft Group, West Palm Beach, Fla. (AFAPL-TR-77-26.)

Cameron, A. (1954) "Surface Failure in Gears," J. Inst. Petrol., vol. 40, 191.

Cameron, A. (1966) The Principles of Lubrication, Wiley, New York.

Cameron, A. and Gohar, R. (1966) "Theoretical and Experimental Studies of the Oil Film in Lubricated Point Contact," Proc. R. Soc. London, Ser. A., 291, 520-536.

Carburi, M. (1777) "Monument Élevé à la Gloire de Pierre-le-Grand, ou Relation Des Travaux et des Moyens Mechaniques Qui ont été employés pour transporter à Petersbourg un Rocher de trois millions pesant, destiné à servir de base à la Statue équestre de cet Empereur; avec un Examen Physique et Chymique de meme Rocher," Paris, (Bookseller: Nyon âiné, Libraine, rue Saint-Lean-de-Beauvois; Printer: Imprimeur-Libraire, rue de la Harpe, vis-à-vis la rue S. Severin).

Castle, P. and Dowson, D. (1972) "A Theoretical Analysis of the Starved Contact," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 131.

Cheng, H. S. (1967) "Calculation of Elastohydrodynamic Film Thickness in High-Speed Rolling and Sliding Contacts," Mechanical Technology Technical Report MTI-67TR24, May 1967.

Cheng, H. S. (1970) "A Numerical Solution to the Elastohydrodynamic Film Thickness in an Elliptical Contact," J. Lubr. Technol., 92(1), 155-162.

Cheng, H. S. and Orcutt, F. K. (1965-66) "A Correlation Between the Theoretical and Experimental Results on the Elastohydrodynamic Lubrication of Rolling and Sliding Contacts," Elastrohydrodynamic Lubrication, Symposium, Leeds, England, Sept. 21-23, 1965, General Papers. Institution of Mechanical Engineers, London, 111-121.

Cheng, H. S. and Sternlicht, B. (1964) "A Numerical Solution for the Pressure, Temperature, and Film Thickness Between Two Infinitely Long, Lubricated Rolling and Sliding Cylinders, Under Heavy Loads," J. Basic Eng. 87(3), 695-707.

Chiu, Y. P. (1974) "An Analysis and Prediction of Lubricant Film Starvation in Rolling Contact Systems," ASME Trans., 17(1), 22-35.

Clark, R. H. (1938) "Earliest Known Ball Thrust Bearing Used in Windmill," English Mechanic, 30 (Dec.) 223.

Coulomb, C. A. (1785) "Théorie des Machines Simples, en ayant égard au frottement de leur parties, et a la roideur des cordages," Academic Royale des Sciences, Mem. Math. Phys., X, Paris, 161-342.

ORIGINAL PAGE IS
OF POOR QUALITY

Crook, A. W. (1957) "Simulated Gear-Tooth Contact: Some Experiments Upon Their Lubrication and Sub-Surface Deformation," Proc. Inst. Mech. Eng., London, 171, 187.

Crook, A. W. (1958) "The Lubrication of Rollers, I," Phil. Trans. R. Soc. London, Ser. A, 250, 387-409.

Crook, A. W. (1961) "Elasto-Hydrodynamic Lubrication of Rollers, Nature," 190, 1182.

Crook, A. W. (1963) "The Lubrication of Rollers, IV - Measurements of Friction and Effective Viscosity," Phil. Trans. R. Soc. London, Ser. A, 255, 281-312.

Dalmaz, G. and Godet, M. (1973) "Traction, Load, and Film Thickness in Lightly Loaded Lubricated Point Contacts," J. Mech. Eng. Sci., 15(6), 400-409.

Dalmaz, G. and Godet, M. (1978) "Film Thickness and Effective Viscosity of Some Fire Resistant Fluids in Sliding Point Contacts," J. Lubr. Technol., 100(2), 304-308.

Denhard, W. G. (1966) "Cost Versus Value of Ball Bearings," Gyro-Spin Axis Hydrodynamic Bearing Symposium, Vol. II, Ball Bearings. Massachusetts Institute of Technology, Cambridge, Mass., Tab. 1.

Desaguliers, J. T. (1734) A Course of Experimental Philosophy, 2 Volumes, London, Volume I, with 32 copper plates.

Dowson, D. (1962) "A Generalized Reynolds Equation for Fluid-Film Lubrication," Int. J. Mech. Sci., 4, 159-170.

Dowson, D. (1965) "Elastohydrodynamic Lubrication - An Introduction and a Review of Theoretical Studies," Institute of Mechanical Engineers, London, Paper R1, 7-15.

Dowson, D. (1968) "Elastohydrodynamics," Proc. Inst. Mech. Eng., London, 182(3A), 151-167.

Dowson, D. (1975) "The Inlet Boundary Condition," Cavitation and Related Phenomena in Lubrication. D. Dowson, M. Godet, and C. M. Taylor, eds., Mechanical Engineering Publications, Ltd., New York, 143-152.

Dowson, D. (1976) "The Origins of Rolling Contact Bearings," T. Sakuri, ed., Proceedings of JSLE-ASLE International Lubrication Conference, Elsevier, Amsterdam, 20-38.

Dowson, D. (1979) History of Tribology, Longman, London and New York.

Dowson, D. (1981) "Lubrication of Joints," Chapter 13 in "The Biomechanics of Joints and Joint Replacements," Edited by D. Dowson and V. Wright, Mechanical Engineering Publications, Bury St. Edmunds, Suffolk. (To be published.)

Dowson, D. and Hamrock, B. J. (1976) "Numerical Evaluation of the Surface Deformation of Elastic Solids Subjected to a Hertzian Contact Stress," ASLE Trans., 19(4), 279-286.

Dowson, D. and Higginson, G. R. (1959) "A Numerical Solution to the Elastohydrodynamic Problem," J. Mech. Eng. Sci., 1(1), 7-15.

Dowson, D. and Higginson, G. R. (1961) "New Roller-Bearing Lubrication Formula," Engineering London, vol. 192, 158.

Dowson, D. and Higginson, G. R. (1964), "A Theory of Involute Gear Lubrication," Institute of Petroleum Gear Lubrication; Proceedings of a Symposium organized by the Mechanical Tests of Lubricants Panel of the Institute, (1964), Elsevier, 8-15.

Dowson, D. and Higginson, G. R. (1966) Elastohydrodynamic Lubrication, The Fundamentals of Roller and Gear Lubrication. Pergamon, Oxford.

ORIGINAL PAGE IS
OF POOR QUALITY

Dowson, D., Saman, W. Y., and Toyoda, S. (1979) "A Study of Starved Elastohydrodynamic Line Contacts," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 92-103.

Dowson, D. and Swales, P. D. (1969) "The Development of Elastohydrodynamic Conditions in a Reciprocating Seal," Proceedings of Fourth International Conference on Fluid Sealing, Vol. 2, Paper 1, British Hydromechanics Research Association, 1-9.

Dowson, D. and Toyoda, S. (1979) "A Central Film Thickness Formula for Elastodydrodynamic Line Contacts." Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 104-115.

Dowson, D. and Whitaker, A. V. (1965-66) "A Numerical Procedure for the Solution of the Elastohydrodynamic Problems of Rolling and Sliding Contacts Lubricated by a Newtonian Fluid," Proc. Inst. Mech. Eng., London, 180(3B), 57.

Dyson, A. (1970) "Flow Properties of Mineral Oils in Elastohydrodynamic Lubrication," Phil. Trans. R. Soc. London, Ser. A, 258(1093), 529-564.

Dyson, A., Naylor, H., and Wilson, A. R. (1965-66) "The Measurement of Oil-Film Thickness in Elastohydrodynamic Contacts," Proceedings of Symposium on Elastohydrodynamic Lubrication, Leeds, England, Institution of Mechanical Engineers, London, 76-91.

Dupuit, A. J. E. J. (1839), "Résumé de Mémoire sur le tirage des voitures et sur le frottement de seconde espece," Competes rendus de l'Académie des Sciences, Paris, IX, 689-700, 775.

ORIGINAL PAGE IS
OF POOR QUALITY

Eaton, J. T. H., ed. (1969) "A Trip Down Memory Lane," The Dragon, XLIV (5), 5-7.

ESDU (1965) "General Guide to the Choice of Journal Bearing Type," Engineering Sciences Data Unit, Item 65007, Institute of Mechanical Engineers, London.

ESDU (1967) "General Guide to the Choice of Thrust Bearing Type," Engineering Sciences Data Unit, Item 67033, Institution of Mechanical Engineers, London.

ESDU (1978) "Grease Life Estimation in Rolling Bearings," Engineering Sciences Data Unit, Item 78032, Institution of Mechanical Engineers, London.

ESDU (1978) "Contact Phenomena. I: Stresses, Deflections and Contact Dimensions for Normally-Loaded Unlubricated Elastic Components," Engineering Sciences Data Unit, Item 78035, Institution of Mechanical Engineers, London.

Evans, H. P., Biswas, S., and Snidle, R. W. (1978) "Numerical Solution of Isothermal Point Contact Elastohydrodynamic Lubrication Problems," Proceeding of First International Conference on Numerical Methods in Laminar and Turbulent Flow, Pentech Press, London, 639-656.

Evans, H. P. and Snidle, R. W. (1978) "Toward a Refined Solution of the Isothermal Point Contact EHD Problem." International Conference Fundmentals of Tribology, Massachusetts Institute of Technology, Cambridge, Mass., June 19-22, 1978.

Fein, R. S. (1968) Discussion on the Papers of J. K. Appeldorn and A. B. Metzner, J. Lubr. Technol., 90, 540-542.

Fellows, T. G., Dowson, D., Perry, F. G., and Plint, M. A. (1963) "Perbury Continuously Variable Ratio Transmission," in N. A. Carter, Ed. Advances in Automobile Engineering, Part 2; Pergamon Press, 123-139.

Foord, C. A., Hammann, W. C., and Cameron, A. (1968) "Evaluation of Lubricants Using Optical Elastohydrodynamics," ASLE Trans., 11, 31-43.

Foord, C. A., Wedeven, L. D., Westlake, F. J. and Cameron, A. (1969-70) "Optical Elastohydrodynamics," Proc. Inst. Mech. Eng., London, Part I, 184, 487-503.

Fromm, H. (1948), "Laminre Strömung Newtonscher und Maxwellscher Slüssigkeiten," Angew Math. Mech., 28(2), 43-54.

Furey, M. J. (1961) "Metallic Contact and Friction Between Sliding Surfaces," ASLE Trans., vol. 4, 1-11.

Gentle, C. R. and Cameron, A. (1973) "Optical Elastohydrodynamics at Extreme Pressure," Nature, 246(5434), 478-479.

Gohar, R. and Cameron A. (1966) "The Mapping of Elastohydrodynamic Contacts," ASLE Trans., 10, 215-225.

Goodman, J. (1912) "(1) Roller and Ball Bearings;" "(2) The Testing of Antifriction Bearing Materials," Proceedings of the Institute of Civil Engineers, CLXXXIX, Session 1911-12, Pt. III, pp. 4-88.

Greenwood, J. A. (1969) "Presentation of Elastohydrodynamic Film-Thickness Results." J. Mech. Eng. Sci., 11(2), 128-132.

Greenwood, J. A. and Kauzlarich, J. J. (1973) "Inlet Shear Heating in Elastohydrodynamic Lubrication," J. Lubr. Technol., 95(4), 401-416.

Grubin, A. N. (1949) "Fundamentals of the Hydrodynamic Theory of Lubrication of Heavily Loaded Cylindrical Surfaces," Investigation of the Contact Machine Components. Kh. F. Ketova, ed., Translation of Russian Book No. 30, Central Scientific Institute for Technology and Mechanical Engineering, Moscow, Chapter 2. (Available from Dept. of Scientific and Industrial Research, Great Britain, Transl. CTS-235, and from Special Libraries Association, Chicago, Trans. R-3554.)

Gunther, R. T. (1930), Early Science in Oxford, Volumes VI and VII, "The Life and Work of Robert Hooke," Vol. VII, Pt. II, 666-679, printed for the author at the Oxford University Press by John Johnson (Oxford).

Hall, L. F. (1957) "A Review of the Papers on the Lubrication of Rotating Bearings and Gears," Proceedings of Conference on Lubrication and Wear, Institution of Mechanical Engineers, pp. 425-429.

Hamilton, G. M. and Moore, S. L. (1971) "Deformation and Pressure in an Elastohydrodynamic Contact," Proc. R. Soc., London, Ser. A, 322, 313-330.

Halling, J. (1976) Introduction of Tribology, Wykeham Publ., London.

Hamrock, B. J. (1976) Elastohydrodynamic Lubrication of Point Contacts, Ph.D. Dissertation, University of Leeds, Leeds, England.

Hamrock, B. J. and Anderson, W. J. (1973) "Analysis of an Arched Outer-Race Ball Bearing Considering Centrifugal Forces," J. Lubr. Technol., 95(3), 265-276.

Hamrock, B. J. and Dowson, D. (1974) "Numerical Evaluation of Surface Deformation of Elastic Solids Subjected to Hertzian Contact Stress," NASA TN D-7774.

Hamrock, B. J. and Dowson, D. (1976a) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part I - Theoretical Formulation," J. Lubr. Technol., 98(2), 223-229.

Hamrock, B. J. and Dowson, D. (1976b) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part II - Ellipticity Parameter Results," J. Lubr. Technol., 98(3), 375-378.

Hamrock, B. J. and Dowson, D. (1977a) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part III - Fully Flooded Results," J. Lubr. Technol., 99(2), 264-276.

Hamrock, B. J. and Dowson, D. (1977b) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part IV - Starvation Results," J. Lubr. Technol., 99(1), 15-23.

Hamrock, B. J. and Dowson, D. (1978) "Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus, Part I - Fully Flooded Conjunction," J. Lubr. Technol., 100(2), 236-245.

Hamrock, B. J. and Dowson, D. (1979a) "Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus, Part II - Starved Conjunction," J. Lubr. Technol., 101(1), 92-98.

Hamrock, B. J. and Dowson, D. (1979b) "Minimum Film Thickness in Elliptical Contacts for Different Regimes of Fluid-Film Lubrication," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechnical Engineering Publications, Ltd., 22-27.

Hardy, W. B. and Doubleday, I. (1922a) "Boundary Lubrication - the Temperature Coefficient," Proc. R. Soc. London, Ser. A, 101, 487-492.

Hardy, W. B. and Doubleday, I. (1922b) "Boundary Lubrication - the Paraffin Series," Proc. R. Soc. London, Ser. A., 100, 550-574.

Harris, T. A. (1966) Rolling Bearing Analysis. Wiley, New York.

ORIGINAL PAGE IS
OF POOR QUALITY

Harris, T. A. (1971) "An Analytical Method to Predict Skidding in Thrust-Loaded, Angular-Contact Ball Bearings," J. Lubr. Technol., vol. 93, 17-24.

Harrison, H. C. (1949) The Story of Sprowston Mill, Phoenix House, London.

Harrison, W. J. (1913) "The Hydrodynamical Theory of Lubrication with Special Reference to Air as a Lubricant," Trans. Cambridge Philos. Soc., xxii (1912-25), 6-54.

Harrison, G. and Trachman, E. G. (1972) "The Role of Compressional Viscoelasticity in the Lubrication of Rolling Contacts," J. Lubr. Technol., 94, 306-312.

Heathcote, H. L. (1921) "The Ball Bearing: In the Making, Under Test, and on Service," Proc. Instn. Automotive Engrs., London, 15, pp. 569-702.

Herrebrugh, K. (1968) "Solving the Incompressible and Isothermal Problem in Elastohydrodynamic Lubrication Through an Integral Equation," J. Lubr. Technol., 90(1), 262-270.

Hersey, M. D. (1966) Theory and Research in Lubrication - Foundations for Future Developments, Wiley, New York.

Hersey, M. S. and Hopkins, R. F. (1954) "Viscosity of Lubricants Under Pressure. Coordinated Data from Twelve Investigations." ASME, New York.

Hertz, H. (1881) "The Contact of Elastic Solids," J. Reine Angew. Math., 92, 156-171.

Hooke, C. J. (1977) "The Elastohydrodynamic Lubrication of Heavily Loaded Contacts," J. Mech. Eng. Sci., 19(4), 149-156.

Hirst, W. and Moore, A. J. (1974) "Non-Newtonian Behavior in Elastohydrodynamic Lubrication," Proc. R. Soc. London, Ser. A, 337, 101-121.

Houghton, P. S. (1976) Ball and Roller Bearings, Applied Science Publishers, Ltd., London.

Jacobson, B. (1970) "On the Lubrication of Heavily Loaded Spherical Surfaces Considering Surface Deformations and Solidification of the Lubricant." Acta Polytech. Scand., Mech. Eng. Ser. No. 54.

Jacobson, B. (1972) "Elasto-Solidifying Lubrication of Spherical Surfaces." American Society of Mechanical Engineers Paper No. 72-LUB-7.

Jacobson, B. (1973) "On the Lubrication of Heavily Loaded Cylindrical Surfaces Considering Surface Deformations and Solidification of the Lubricant," J. Lubr. Technol., 95(3), 321-27.

Jamison, W. E., Lee, C. C., and Kauzlarich, J. J. (1978) "Elasticity Effects on the Lubrication of Point Contacts," ASLE Trans., 21(4), 299-306.

Johnson, B. L. (1964) "A 'Stainless High Speed' Steel for Aerospace Applications," Metal Prog., 86(3), 116-118.

Johnson, B. L. (1965) "High Temperature Wear Resisting Steel," U.S. Patent No. 3,167,423, Jan. 1965.

Johnson, K. L. (1970) "Regimes of Elastohydrodynamic Lubrication." J. Mech. Eng. Sci., 12(1), 9-16.

Johnson, K. L. and Cameron, R. (1967) "Shear Behavior of Elastohydrodynamic Oil Films at High Rolling Contact Pressures," Proc. Ins. Mech. Eng., Part 1, 182, 307-319.

Johnson, K. L. and Roberts, A. D. (1974) "Observation of Viscoelastic Behaviour of an Elastohydrodynamic Lubricant Film," Proc. R. Soc. London, Ser. A, 337, 217-242.

Johnson, K. L. and Tevaarwerk, J. L. (1977) "Shear Behaviour of Elastohydrodynamic Oil Films," Proc. R. Soc. London, Ser. A, 356, 215-236.

ORIGINAL PAGE IS
OF POOR QUALITY

Jones, A. B. (1946) "Analysis of Stresses and Deflections," New Departure Engineering Data, General Motors Corp., Bristol, Conn.

Jones, A. B. (1956) "The Mathematical Theory of Rolling-Element Bearings," Mechanical Design and Systems Handbook.

Kannel, J. W., Bell, J. C., and Allen, C. M. (1964) "Methods for Determining Pressure Distribution in Lubricated Rolling Contact," ASLE Paper 64-LC-23, Presented at ASME-ASLE Lubrication Conference, Washington, D.C., Oct. 13-16, 1964.

Kakuta, K. (1979) "The State of the Art of Rolling Bearings in Japan," Bull. Japan Soc. Prec. Eng., 13(4), 169-176.

Kapitza, P. L. (1955) "Hydrodynamic Theory of Lubrication During Rolling," Zh. Tekh. Fiz., 25(4), 747-762.

Koye, K. A. and Winer, W. O. (1980) " An Experimental Evaluation of the Hamrock and Dowson Minimum Film Thickness Equation for Fully Flooded EHD Point Contacts," International ASME/ASLE Lubrication Conference, San Francisco, August 1980.

Kunz, R. K. and Winer, W. O. (1977) Discussion 275-276, to Hamrock, B. J. and Dowson, D. "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part III - Fully Flooded Results," J. Lubr. Technol., 99(2), 264-275.

Lane, T. B. (1951) "Scuffing Temperatures of Boundary Lubricant Films," Br. J. Appl. Phys., 2, (Suppl. 1), 35-38.

Lane, T. B. and Hughes, J. R. (1952) "A Study of the Oil Film Formation in Gears by Electrical Resistance Measurements," Br. J. Appl. Phys., 3(10), 315-318.

Lamb, H. (1932) Hydrodynamics. Cambridge University Press.

Layard, A. H. (1849) Nineveh and Its Remains, Vols. I and II, John Murray, London.

Layard, A. H. (1853) Discoveries in the Ruins of Nineveh and Babylon, Vols. I and II, John Murray, London.

Lee, D., Sanborn, D. M., and Winer, W. O. (1973) "Some Observations of the Relationship Between Film Thickness and Load in High Hertz Pressure Sliding Elastohydrodynamic Contacts," J. Lubr. Technol., 95(3), 386.

Leibnitz, G. W. (1706) "Tentamen de natura et remedlie resistenziarum in machines," Miscellanea Berolinensia. Class. mathem, 1710, (Jean Boudot, Paris), 1, 307.

Lewicki, W. (1955) "Some Physical Aspects of Lubrication in Rolling Bearings and Gears," Engineer, 200 (5193), 176-178, and (5194), 212-215.

Lundberg, G. and Palmgren, A. (1947) "Dynamic Capacity of Rolling Bearings," Acta Polytech., Mech. Eng. Sci., 1(3).

Martin, H. M. (1916) "Lubrication of Gear Teeth," Engineering, London, 102, 199.

McEwen, E. (1952) "The Effect of Variation of Viscosity with Pressure on the Load Carrying Capacity of Oil Films Between Gear Teeth," J. Inst. Petrol., 38, 646.

Meldahl, A. (1941) "Contribution to the Theory of the Lubrication of Gears and of the Stressing of the Lubricated Flanks of Gear Teeth," Brown Boveri Review, 28(11), 374.

Merritt, H. E. (1935) "Worm-Gear Performance," Proc. Inst. Mech. Eng., London, 129, 127-158.

Meyer, D. R. and Wilson, C. C. (1971) "Measurement of Elastohydrodynamic Oil Film Thickness and Wear in Ball Bearings by the Strain Gage Method," J. Lubr. Technol., 93(2), 224-230.

Moes, H. (1965-66) "Communication, Elastohydrodynamic Lubrication," Proc. Inst. Mech. Eng., London, 180(3B), 244-245.

Moes, H. and Bosma, R. (1972) "Film Thickness and Traction in EHL at Point Contact," Proceedings of Second Symposium on Elastohydrodymamic Lubrication, Leeds, England, Institution of Mechanical Engineers, London, 149.

Moore, A. J. (1973) "Non-Newtonian Behaviour in Elastohydrodynamic Lubrication," Ph.D. Thesis, University of Reading.

Morgan, M. H. and Warren, H. L. (1960) Translation of Vitruvius: The Ten Books of Architecture, Dover, New York.

Morin, A. J. (1835) "Nouvelles expériences faites à Metz en 1833 sur le frottement, sur la transmission due movement par le choc, sur le résistance des milieun imparfaits a le pénétration des projectiles, et sur le frottement pendant le choc," Mem. Savans Etrang. (Paris), VI, 641-785; Ann. Min. X, (1836), 27-56.

Nagaraj, H. S., Sanborn, D. M., and Winer, W. O. (1977) "Effects of Load, Speed, and Surface Roughness on Sliding EHD Contact Temperature," J. Lubr. Technol., 99(4), 254-263.

Navier, C. L. M. H. (1823) "Memoire sur les lois du mouvement des fluides," Mem. Acad. R. Sci., 6(2), 389-440.

Needham, J. (1965) Science and Civilization in China, Vol. 4, Physics and Physical Technology, Part II, Mechanical Engieering, Cambridge University Press.

Newton, I. (1687) Philosophiae Naturales Principia Mathematica, Imprimature S. Pepys, Reg. Soc. Praeses, 5 Julii 1686. Revised and supplied with a historical and explanatory appendix by F. Cajori, edited by R. T. Crawford (1934), and published by the University of California Press, Berkeley and Los Angeles (1966).

Orcutt, F. K. and Cheng, H. S. (1966) "Lubrication of Rolling-Contact Instrument Bearings," Gyro-Spin Axis Hydrodynamic Bearing Symposium, Vol. 2, Ball Bearings, Massachusetts Institute of Technology, Cambridge, Mass., Tab. 5.

Pai, S. I (1956) Viscous Flow Theory, Vol. I - Laminar Flow. Van Nostrand Reinhold, New Jersey.

Palmgren, A. (1945) "Ball and Roiller Bearing Engineering," S. K. F. Industries, Philadelphia.

Parker, R. J. and Hodder, R. S. (1978) "Roller-Element Fatigue Life of AMS 5749 Corrosion Resistant, High Temperature Bearing Steel," J. Lubr. Technol., 100(2), 226-235.

Parker, R. J. and Kannel, J. W. (1971) "Elastohydrodynamic Film Thickness Between Rolling Disks with a Synthetic Paraffinic Oil to 589 K (600° F); NASA TN D-6411.

Parker, R. J. and Zaretsky, E. V. (1978) "Rolling-Element Fatigue Life of AISI M-50 and 18-4-1 Balls." NASA TP-1202.

Peppler, W. (1936) "Untersuchunge uber die Drukubertragung bei Balasteten und Geschmierten um Laufenden Achsparallelen Zylinder," Maschinenelemente-Tagung Archen 1935, 42; V. D. I. Verlag, Berlin, 1936.

Peppler, W. (1938) "Druchubertragung an Geschmeirten Zylindriachen Gleit und Wälzflächen," V. D. I. Forschungshaft, 391.

Petrov, N. P. (1883) "Friction in Machines and the Effect of the Lubricant," Inzh. Zh., St. Peterb., 1, 71-140; 2, 227-279; 3, 377-436; 4, 535-564.

Petrusevich, A. S. (1951) "Fundamental Conclusion from the Contact-Hydrodynamic Theory of Lubrication," dzo. Akad. Nauk. SSSR (OTN), 2, 209.

Piggott, S. (1968) "The Earliest Wheeled Vehicles and the Caucasian Evidence," Proc. Prehist. Soc., XXXIV, (8), 266-318.

Pirvics, J. (1980) "Numerical Analysis Techniques and Design Methology for Rolling Element Bearing Load Support Systems," in International Conference on Bearing Design: Historical Aspects, Present Technology and Future Problems; Century 2 - Emerging Technology, W. J. Anderson, ed., American Society of Mechanical Engineers, New York, 1980, 47-85.

Plint, M. A. (1967) "Traction in Elastohydrodynamic Contact," Proc. Inst. Mech. Eng., London, Part 1, 182(14), 300-306.

Poritsky, H., Hewlett, C. W., Jr., and Coleman, R. E., Jr. (1947) "Sliding Friction of Ball Bearings of the Pivot Type," J. Appl. Mech., 14(4), 261-268.

Pritchard, C. (1981) "Traction Between Rolling Steel Surfaces - A Survey of Railway and Laboratory Services," Proceedings of the 7th Leeds-Lyon Symposium on 'Friction and Traction, Leeds, September 1980, Mechanical Engineering Publications. (To be published.)

Ramelli, A. (1588) "Le Diverse et Artificiose Machine," Paris, France.

Ranger, A. P., Ettles, C. M. M., and Cameron, A. (1975) "The Solution of Point Contact Elastohydrodynamic Problem," Proc. R. Soc. London, Ser. A, 346, 277-244.

Reti, L. (1971) "Leonardo on Bearings and Gears," Scientific American, 224, (2), 101-110.

Reynolds, O. (1875) "On Rolling Friction," Phil. Trans. R. Soc., 166, Pt. 1, 155.

Reynolds, O. (1886) "On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil," Philos. Trans. R. Soc. London, 177, 157-234.

Roelands, C. J. A. (1966) Correlational Aspects of the Viscosity-Temperature-Pressure Relationship of Lubricating Oils. Druk. V. R. B., Groningen, Netherlands.

Rowe, J. (1734) "All Sorts of Wheel-Carriage Improved," printed for Alexander Lyon under Tom's Coffee House in Russell Street, Covent Garden, London.

Sanborn, D. M. (1969) "An Experimental Investigation of the Elastohydrodynamic Lubrication of Point Contacts in Pure Sliding," Ph.D. Thesis, University of Michigan.

Schlatter, R. (1974) "Double Vacuum Melting of High Performance Bearing Steels," Ind. Heat. 41(9), 40-55.

Shaw, M. C. and Macks, E. F. (1949) Analysis and Lubrication of Bearings, McGraw-Hill, New York.

Sibley, L. B., Bell, J. C., Orcutt, F. K., and Allen, C. M. (1960) "A Study of the Influence of Lubricant Properties on the Performance of Aircraft Gas Turbine Engine Rolling Contact Bearings," WADD Technical Report, 60-189.

Sibley, L. B. and Orcutt, F. K. (1961) "Elasto-Hydrodynamic Lubrication of Rolling Contact Surfaces," Trans. Amer. Soc. Lub. Engrs., 4(2), 234.

Smith, F. W. (1959) "Lubricant Behavior in Concentrated Contact Systems - The Caster Oil - Steel System," Wear, 2(4), 250-264.

ORIGINAL PAGE IS
OF POOR QUALITY

Smith, F. W. (1962) The Effect of Temperature in Concentrated Contact Lubrication. ASLE Trans. 5(1), 142-148.

Stokes, G. G. (1845) "On the Theory of Internal Friction of Fluids in Motion," Trans. Cambridge Philos. Soc. 8, 287-319.

Stribeck, R. (1901) " Kugellager fur beliebige Belastungen," Z. Ver. dt. Ing., 45(3), 73-125.

Stribeck, R (1907) "Ball Bearings for Various Loads" - translation by H. Hess, Trans. Am. Soc. Mech. Engrs., 29, 420.

Swingler, C. L. (1980) "Surface Roughness in Elastohydrodynamic Line Contacts," Ph.D. Thesis, University of London (Imperial College).

Tabor, D. (1962) "Introductory Remarks," in Rolling Contact Phenomena, J. B. Bidwell, ed., Elsevier, Amsterdam, 1-5.

Tallian, T. E. (1969) "Progress in Rolling Contact Technology," Report AL 690007, SKF Industries, King of Prussia, Pa.

Tallian, T., Sibley, L., and Valori, R. (1965) "Elastohydrodynamic Film Effects on the Load-Life Behavior of Rolling Contacts," ASMS Paper 65-LubS-11.

Theyse, F. H. (1966) "Some Aspects of the Influence of Hydrodynamic Film Formation on the Contact Between Rolling/Sliding Surfaces," Wear, 9, 41-59.

Thorp, N. and Gohar, R. (1972) "Oil Film Thickness and Shape for a Ball Sliding in a Grooved Raceway," J. Lubr. Technol., 94(3), 199-210.

Timoshenko, S. and Goodier, J. N. (1951) Theory of Elasticity, 2nd ed., McGraw-Hill, New York.

Trachman, E. G. and Cheng, H. S. (1972) "Thermal and Non-Newtonian Effects on Traction in Elastohydrodynamic Contacts," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 142-148.

Tower, B. (1883) "First Report on Friction Experiments (Friction of Lubricated Bearings)," Proc. Inst. Mech. Eng., London, 632-659.

Turchina, V., Sanborn, D. M., and Winer, W. O. (1974) "Temperature Measurements in Sliding Elastohydrodynamic Point Contacts," J. Lubr. Technol., 96(3), 464-471.

Ucelli, G. (1940) "Le Navi Di Nemi," La Liberia Dello Stato, Roma.

Valori, R. (1978) Discussion to Parker, R. J. and Hodder, R. S. (1978) Rolling-Element Fatigue Life of AMS 5749 Corrosion Resistant, High Temperature Bearing Steel," J. Lubr. Technol., 100(2), 226-235.

Van Natrus, L., Polly, J., and Van Vuuren, C. (1734 and 1736), Groot Volkomen Moolenbock, 2 Volumes, Amsterdam.

Varlo, C. (1772) "Reflections Upon Friction with a Plan of the New Machine for Taking It Off in Wheel-Carriages, Windlasses of Ships, etc., Together with Metal Proper for the Machine, the Full Directions for Making It."

Vaughan, P. (1794) "Axle Trees, Arms, and Boxes," British Patent No. 2006 of A.D. 1794, 1-2, accompanied by 11 diagrams on one sheet.

Wailes, R. (1954) The English Windmill, Routledge & Kegan Paul, London.

Wailes, R. (1957) "Windmills" in History of Technology, C. Singer, E. J. Holmyard, A. R. Hall, and T. I. Williams, eds., Volume III, Oxford University Press, pp. 89-109.

Weber, C. and Saalfeld, K. (1954) Schmierfilm bei Walzen mit Verformung, Zeits ang. Math. Mech. 34 (Nos. 1-2).

Wedeven, L. E., Evans, D., and Cameron, A. (1971) "Optical Analysis of Ball Bearing Starvation," J. Lubr. Technol., 93(3), 349-363.

Weibull, W. (1949) "A Statistical Representation of Fatigue Failures in Solids," Trans. Roy. Inst. Technol., (27), Stockholm.

Whomes, T. L. (1966) The Effect of Surface Quality of Lubricating Film Performance, Ph.D. Dissertation, University of Leeds, Leeds, England.

Wilcock, D. F. and Booser, E. R. (1957) Bearing Design and Application. McGraw-Hill, New York.

Willis, T., Seth, B., and Dave, M. (1975) "Evaluation of a Diffraction Method for Thickness Measurement of Oil-Filled Gaps," J. Lubr. Technol. 97(4), 649-650.

Wilson, A. R. (1979) "The Relative Thickness of Grease and Oil Films in Rolling Bearings," Proc. Inst. Mech. Eng., London, 193(17), 185-192.

Winn, L. W., Eusepi, M. W., and Smalley, A. J. (1974) "Small, High-Speed Bearing Technology for Cryogenic Turbo-Pumps," MTI-74TR29, Mechanical Technology, Inc., Latham, N.Y. (NASA CR-134615.)

Wolveridge, P. E., Baglin, K. P., and Archard, J. G. (1971) "The Starved Lubrication of Cylinders in Line Contact," Proc. Inst. Mech. Eng., London, 185(1), 1159-1169.

Zaretsky, E. V., Anderson, W. J., and Bamberger, E. N. (1969) "Rolling Element Bearing Life for 400° to 600° F." NASA TN D-5002.

Zaretsky, E. V., Parker, R. J., and Anderson, W. J. (1967) "Component Hardness Differences and Their Effect on Bearing Fatigue," J. Lubr. Technol., 87(1), 47-62.

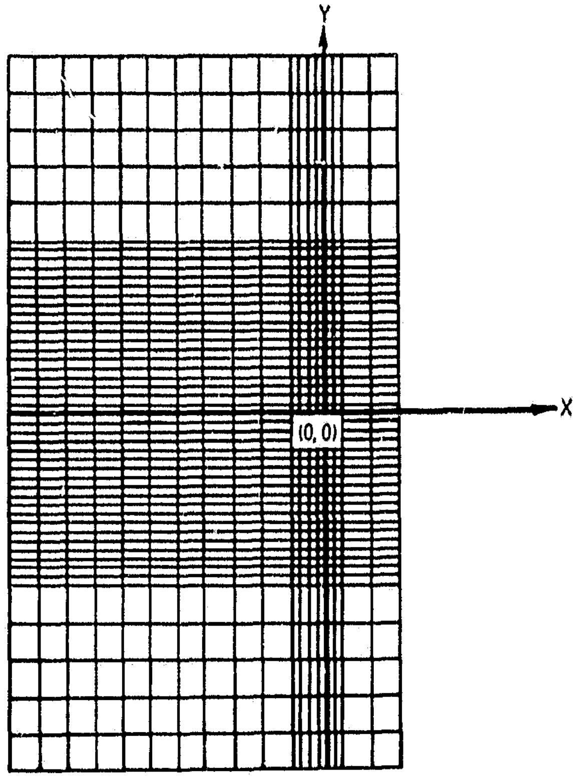


Figure 6.1. - Variable nodal structure used for numerical calculations.

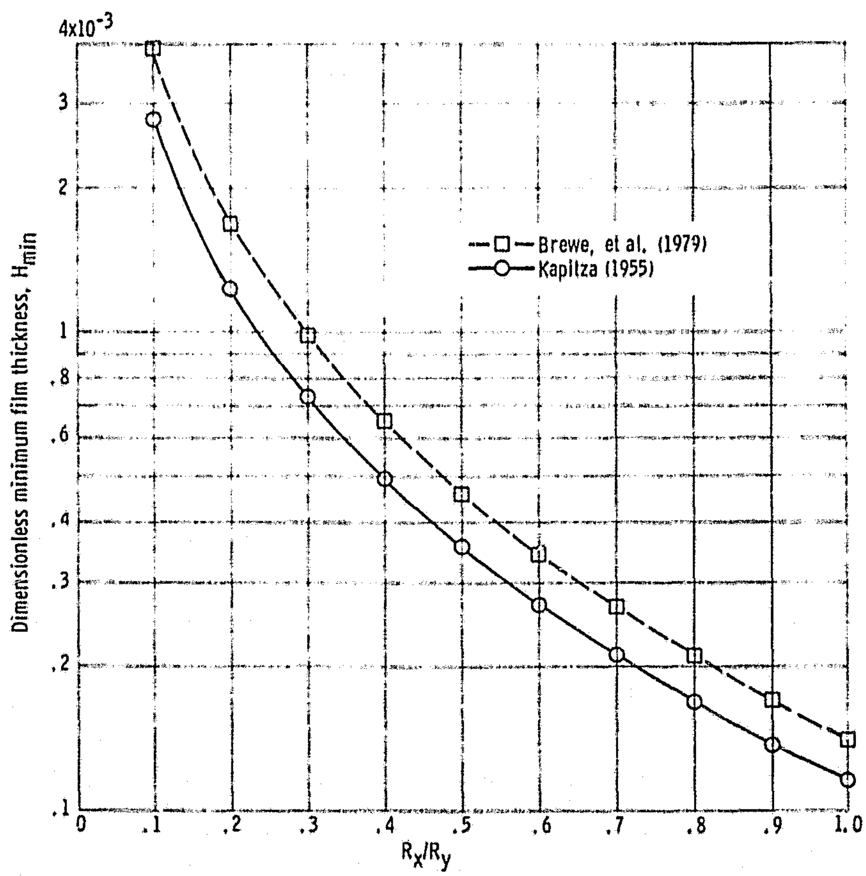


Figure 6.2. - Film thickness results of Kapitza (1955) and Brewe, et al. (1979) for range of R_x/R_y and $U/W = 1x10^{-3}$.

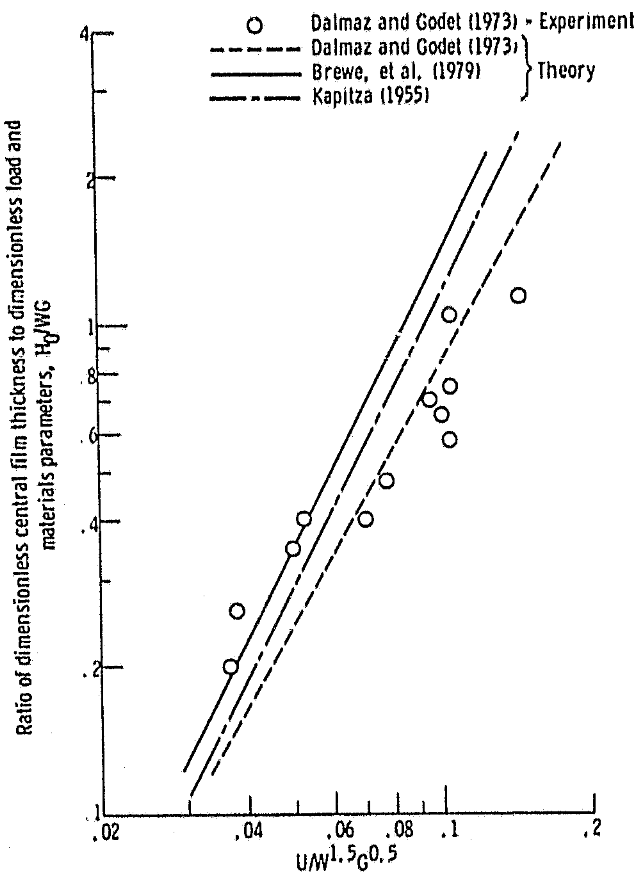
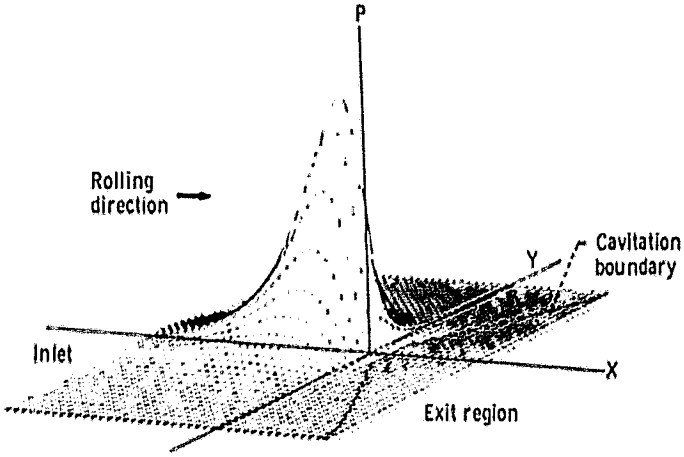
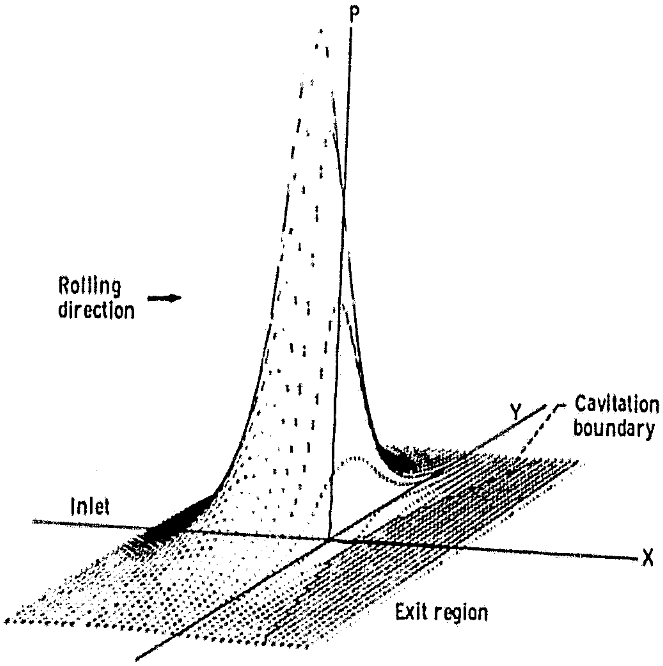


Figure 6.3. - Theoretical and experimental results.

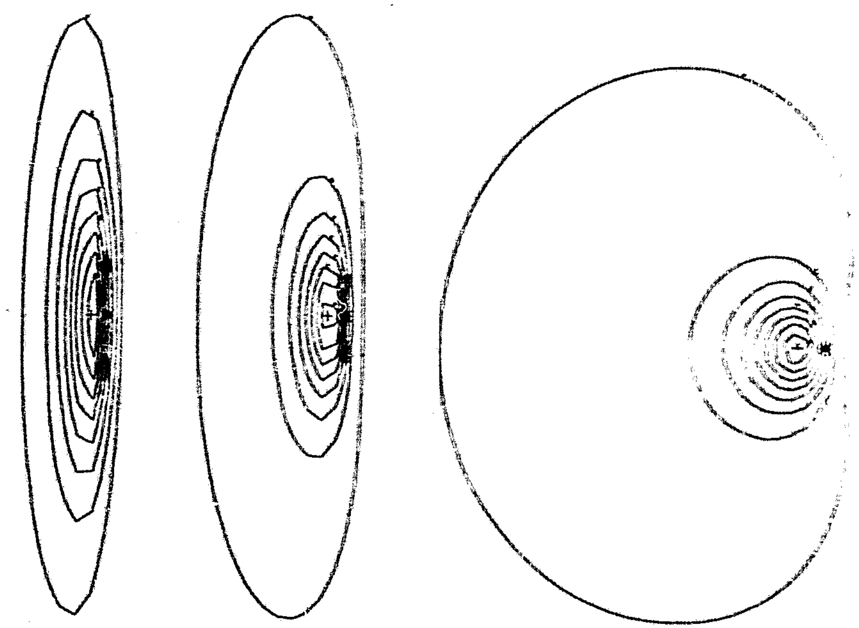


(a) Radius ratio, α_a, 1.00.



(b) Radius ratio, α_a, 36.54.

Figure 6.4. - Three-dimensional representations of pressure distributions as viewed from exit region, illustrating cavitation boundary.



(a) α_a = 25.29. (b) α_a = 8.30. (c) α_a = 1.00.

Figure 6.5. - Pressure contours for three radius ratios.